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Mean Square Stability and Performance Study for Wireless Networked Control System with Multiple Stochastic Packet Loss

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***Abstract*— This paper proposes a novel H∞ control law for Wireless Networked Control System, ensuring both robustness and performance. It is founded on the design of a dynamic output feedback controller for a class of uncertain linear discrete-time system that are subject to external perturbations, model-related uncertainties and network data loss. In fact, the model-related uncertainties, in both control and state matrix, are considered as parametric uncertainties, while the packets loss are occurring randomly in both sensor-to-controller and controller-to-actuator channels. In accordance with the wireless network stochastic behaviour, we modelled the random multiple packet loss as an independent binary Bernoulli white sequence and the uncertainties in the state and control matrix as norm-bounded uncertainty. The designed dynamic controller achieve the closed loop stability of the studied system, in the sense of the exponential mean square convergence, and deal with the H∞ disturbance attenuation.**

The suboptimal controller parameters are derived based on a feasibility LMI (Linear Matrix Inequality) condition. Numerical simulation is performed, to prove the efficiency of the proposed theoretical approach.

***Keywords*— Wireless Networked Control Systems (WNCS) – Robust control – Packet dropout – Linear Matrix Inequality (LMI) – Norm bounded uncertainty – Dynamic state feedback controller.**

I. INTRODUCTION

The Networked Control Systems (NCSs) are a distributed components (actuators, sensors and controllers) which are interconnected by a shared limited bandwidth network medium.

Knowing that the physical wires networks are expansive and hard to maintain specially in the large networks, the insertion of the wireless network in the control loop provides a large benefits in terms of costs, maintenance, implementation and scalability. To take advantage of the flexibility of the wireless network, the NCS models must address the wireless network induced data loss, congestion, transmission delay and interference.

The problem of these NCSs are tackled based on two key points: the first one involve the network handling in order to ensure the required process Quality of Service by providing a shared communication medium that meets the bandwidth requirement, the transmission time optimization and the required packet loss thresholds. The second one focuses on the process handling in order to guarantee the Quality of Control; it consist on modelling a control laws that deals with transmission delay, disturbance, uncertainty and packet loss.

In fact, when addressing the data dropouts, researchers predict this network induced imperfection as one of these two models : the first one consider the data dropout as Markov chains [13], the second one as a binary distributed white sequence that follow the Bernoulli law [12][14] [15].

Due to the expansion of the wireless communication, researchers focuses on reaching the network control via wireless networks in order to support emerging media-rich application, to deliver more quality of services to more users in less time and to achieve benefits in terms of reliability, wiring (less cabling), implementation (handling multiple geographically distributed nodes) costs and maintenance (efficient monitoring and less restrictive control action over a distributed computation).

Moreover, the WNCS are considered as a very flexible NCS since, first they could be easily implemented even on resource constrained or low-power node and second, their design could be extended by adding new subsystems.

Further, Wireless NCS are challenging researchers because transmission between neighbour nodes depends on (1) collisions that might happen when nodes try to transmit at the same time, (2) the amount of power used to send data, (3) the used protocol.

The main standards protocols in wireless communication are: Wireless Local Area Networks (WLAN/WiFi 802.11), Wireless Personal Area Networks (ZigBee 802.15) and Wireless Metropolitan Area Networks (WiMAX).

The effectiveness of NCSs was proven in many domains such as Swarm Robotics, Multi-Camera Real Time Tracking, Internet and transportation [1] [2] [3] and Mobile Sensor Networks [4].

The contribution of this paper is a study of a H∞ control approach for a class of uncertain linear discrete-time Wireless Networked Control Systems that are subject to stochastic model-related uncertainty and multiple packet dropouts. This

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scheme is based on the design of a dynamic feedback controller that exponentially stabilize the WNCS in the sense of the mean square convergence and robustly attenuate the perturbation.

Packets dropouts are modelled, in both measurement and control channels, as binary white sequence parameter that obey to the Bernoulli law and the model-related uncertainties, in both control and state matrix as tackled as a parametric norm-bounded uncertainties.

The paper sections are presented as follows: Section 1 introduce the topic purpose. Section 2 provides the system description in a mathematical representation. Section 3 tackle the problem formulation. Section 4 demonstrate the stability analysis, the performance study and the dynamic feedback controller design of the closed loop system. Section 4 prove the accuracy of the described approach, using a numerical example, to achieve the exponential mean square stability and to improve the H∞ disturbance attenuation of the studied system.

II. System description

Being considered the WNCS with time-varying packet dropouts, the state space representation of the plant is described by:

$$\begin{cases} x_{k+1} = (A+\Delta A)x_k + (A+\Delta B)u_k + B_w w_k \\ ys_k = Cx_k + C_w w_k \\ z_k = Dx_k \end{cases} \tag{1}$$

Where $x_k \in \mathrm{R}^n$ is the state signal, $u_k \in \mathrm{R}^m$ is the control input signal, $w_k \in \mathrm{R}^q$ is the disturbance input signal, $ys_k \in \mathrm{R}^p$ is the measurement output signal and $z \in \mathrm{R}^r$ is the controlled output signal.

The parameter uncertainties ΔA, ΔB are described as:

$$\begin{cases} \|\Delta \mathrm{A}\| \le \rho_a \\ \|\Delta \mathrm{B}\| \le \rho_b \end{cases} \tag{2}$$

We consider, the measurement with packet loss and the control input sent over the network as:

$$\begin{cases} y_k = (1-\beta)ys_k + \beta y_{k-1} \\ u_k = (1-\alpha)uc_k + \alpha u_{k-1} \end{cases} \tag{3}$$

Where $y_k \in \mathrm{R}^p$ is the output signal, $uc_k \in \mathrm{R}^m$ is the control signal and α and β; are a mutually independent random variables that follow the binary distributed white sequence of Bernoulli [5]:

$$Prob(x=i) = \begin{cases} \mathrm{E}(x) = \bar{x} & for\ \ i=0 \\ 1-\mathrm{E}(x) = 1-\bar{x} & for\ \ i=1 \end{cases} \tag{4}$$

The NCS representation for uncertain system with data loss is derived from *(1) to (4)* and given by:

$$\begin{cases} x_{k+1} = (A+\Delta \mathrm{A})x_k + (B+\Delta \mathrm{B})u_k + B_w w_k \\ ys_k = Cx_k + C_w w_k \\ u_k = (1-\alpha)uc_k + \alpha u_{k-1} \\ y_k = (1-\beta)ys_k + \beta y_{k-1} \end{cases} \tag{5}$$

III. Problem formulation

In order to study the H∞ stability of the above uncertain system, we have defined for the augmented form of the system (6) and the dynamic controller (7) such as:

$$\begin{cases} \tilde{x}_{k+1} = \tilde{A}\tilde{x}_k + \tilde{B}uc_k + \tilde{B}_w w_k \\ y_k = \tilde{C}\tilde{x}_k + \tilde{C}_w w_k \\ z_k = \tilde{D}\tilde{x}_{k+1} \end{cases} \tag{6}$$

$$\begin{cases} \hat{x}_{k+1} = A_{ctr}\hat{x}_k + B_{ctr}y_k \\ uc_k = C_{ctr}\hat{x}_k \end{cases} \tag{7}$$

Where:

$$\tilde{x}_{k+1} = \begin{bmatrix} x_k \\ u_{k-1} \\ y_{k-1} \end{bmatrix}; \tilde{A} = \begin{bmatrix} A+\Delta A & \alpha(B+\Delta \mathrm{B}) & 0 \\ 0 & \alpha I & 0 \\ (1-\beta)C & 0 & \beta I \end{bmatrix};$$

$$\tilde{B} = \begin{bmatrix} (1-\alpha)(B+\Delta \mathrm{B}) \\ (1-\alpha)I \\ 0 \end{bmatrix}; \tilde{C} = \begin{bmatrix} (1-\beta)C & 0 & \beta I \end{bmatrix};$$

$$\tilde{C}_w = (1-\beta)C_w; \tilde{D} = \begin{bmatrix} D & 0 & 0 \end{bmatrix}; \tilde{B}_w = \begin{bmatrix} B_w \\ 0 \\ (1-\beta)C_w \end{bmatrix}$$

A_{ctr}, B_{ctr} and C_{ctr} are the controller matrices to be defined. For that purpose, we define the uncertain discrete time-varying system *(8)* with stochastic parameters α and β:

$$\begin{cases} X_{k+1} = \begin{bmatrix} \tilde{x}_{k+1} \\ \hat{x}_{k+1} \end{bmatrix} = \begin{bmatrix} \tilde{A} & \tilde{B}C_{ctr} \\ B_{ctr}\tilde{C} & A_{ctr} \end{bmatrix} X_k + \begin{bmatrix} \tilde{B}_w \\ B_{ctr}\tilde{C}_w \end{bmatrix} w_k \\ z_k = \begin{bmatrix} \tilde{D} & 0 \end{bmatrix} \ddot{X}_k \end{cases} \tag{8}$$

We carry on with the division of stochastic parameters into stochastic section and deterministic section, we infer:

$$\begin{cases} X_{k+1} = (\bar{A}+\overline{\Delta A})X_k + \bar{B}_W w_k + (\beta-\bar{\beta})\bar{B}_\beta w_k \\ \qquad + \left((\alpha-\bar{\alpha})(\bar{B}+\overline{\Delta B}) + (\beta-\bar{\beta})\bar{A}_\beta\right)X_k \\ z_k = \bar{D}X_k \end{cases} \tag{9}$$

Where:

$$\bar{A} = \begin{bmatrix} A_1 & B_1C_{ctr} \\ B_{ctr}C_1 & A_{ctr} \end{bmatrix}, \bar{B} = \begin{bmatrix} B_2 & -B_3C_{ctr} \\ 0 & 0 \end{bmatrix}$$

$$\overline{\Delta A} = \begin{bmatrix} \Delta A_1 & \Delta B_1C_{ctr} \\ 0 & 0 \end{bmatrix}, \overline{\Delta B} = \begin{bmatrix} \Delta B_2 & -\Delta B_3C_{ctr} \\ 0 & 0 \end{bmatrix}$$

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$$\bar{A}_\beta = \begin{bmatrix} A_\beta & 0 \\ B_{ctr}C_\beta & 0 \end{bmatrix}, \bar{B}_W = \begin{bmatrix} B_{1w} \\ B_{ctr}C_{1w} \end{bmatrix},$$

$$\bar{B}_\beta = \begin{bmatrix} B_{\beta w} \\ -B_{ctr}C_w \end{bmatrix}, \bar{D} = \begin{bmatrix} D & 0 \end{bmatrix}$$

$$A_1 = \begin{bmatrix} A & \bar{\alpha}B & 0 \\ 0 & \bar{\alpha}I & 0 \\ \left(1-\bar{\beta}\right)C & 0 & \bar{\beta}I \end{bmatrix}; B_1 = \begin{bmatrix} \left(1-\bar{\alpha}\right)B \\ \left(1-\bar{\alpha}\right)I \\ 0 \end{bmatrix};$$

$$C_1 = \begin{bmatrix} \left(1-\bar{\beta}\right)C & 0 & \bar{\beta}I \end{bmatrix}; C_\beta = \begin{bmatrix} -C & 0 & I \end{bmatrix}$$

$$B_2 = \begin{bmatrix} 0 & B & 0 \\ 0 & I & 0 \\ 0 & 0 & 0 \end{bmatrix}; B_3 = \begin{bmatrix} B \\ I \\ 0 \end{bmatrix}; \Delta B_2 = \begin{bmatrix} 0 & \Delta B & 0 \\ 0 & I & 0 \\ 0 & 0 & 0 \end{bmatrix};$$

$$\Delta B_3 = \begin{bmatrix} \Delta B \\ 0 \\ 0 \end{bmatrix}; A_\beta = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ -C & 0 & \mathrm{I} \end{bmatrix}; B_{\beta w} = \begin{bmatrix} 0 \\ 0 \\ -C_w \end{bmatrix};$$

$$B_{1w} = \begin{bmatrix} B_w \\ 0 \\ \left(1-\bar{\beta}\right)C_w \end{bmatrix}; C_{1w} = \left(1-\bar{\beta}\right)C_w$$

$$B_\alpha = \begin{bmatrix} B \\ I \\ 0 \end{bmatrix}; A_\beta = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ -C & 0 & \mathrm{I} \end{bmatrix}; B_\beta = \begin{bmatrix} 0 \\ 0 \\ -C_w \end{bmatrix};$$

IV. STABILITY ANALYSIS AND CONTROL DESIGN

In order to study the stability of the uncertain stochastic system *(9)* we proceed with the design of the controller (7) such that the closed loop system satisfies the two requirements mentioned here below:

1- The H∞ mean square stochastic stability of the closed loop system.

The H∞ performance disturbance attenuation under zero initial condition.

A. The H∞ exponential mean square stability

The following theorem gives sufficient condition for the H∞ mean square stochastic stability for the studied WNCS.

Theorem 1*:* The system *(9)* is mean square stable if there exist a positive matrix P satisfying the LMI (10).

Proof: by defining the Lyapunov function $V_k = X_k^T P X_k$ where P is a unique positive definite matrix, it follows from *(9)* that:

$$\begin{aligned} &\mathrm{E}\left\{V\left(X_{k+1}\right) | X_k, .., X_0\right\} - V\left(X_k\right) \\ &= \mathrm{E}\left\{X_{k+1}^T P X_{k+1}\right\} - X_k^T P X_k = X_k{}^T \Phi\, X_k \end{aligned} \tag{11}$$

Where:

$$\begin{aligned} \Phi = &\left(\bar{A} + \bar{\Delta}A\right)^T Q\left(\bar{A} + \bar{\Delta}A\right) + \left(\bar{\beta} - \bar{\beta}^2\right)\bar{A}_\beta{}^T Q\bar{A}_\beta \\ &+ \left(\bar{\alpha} - \bar{\alpha}^2\right)\left(\bar{B} + \bar{\Delta}B\right)^T Q\left(\bar{B} + \bar{\Delta}B\right) - Q \end{aligned} \tag{12}$$

By applying Schur complement, we conclude that $\Phi < 0$ if and only if (10) holds.

$\Phi < 0$ Then:

$$X_k{}^T \Phi\, X_k \le -\theta(-\Phi) X_k{}^T X_k < -\delta X_k{}^T X_k < -\frac{\delta}{\gamma} V\left(X_k\right) \tag{13}$$

Where $0 < \delta < \min(\theta_{\min}(-\Phi), \theta_{\max}(P))$

From (13), we infer that:

$$\mathrm{E}\left\{V\left(X_{k+1}\right) | X_k, X_{k-1}, \ldots, X_0\right\} \le \left(1 - \frac{\delta}{\gamma}\right)^k V\left(X_0\right) \tag{14}$$

From (14) and using results in [6] we come to the conclusion that the system (9) is mean square stable. Then the proof is accomplished.

B. The H∞ controller design

In this section we study the H∞ performance which is the disturbance attenuation of the closed loop system (9). The main result is given in the theorem 2.

Theorem 2*:* The controller (7) is designed so that the closed loop system (9) is exponentially mean square stable and the robust disturbance attenuation, under zero initial condition, is achieved if there exists a positive definite matrix P and a scalar $\varphi > 0$ satisfying LMI (15).

Where $\surd_\alpha = \sqrt{\left(\bar{\alpha} - \bar{\alpha}^2\right)}$ and $\surd_\beta = \sqrt{\left(\bar{\beta} - \bar{\beta}^2\right)}$

Proof: if for any non-zero w_k, we define:

$$\mathrm{E}\left\{V\left(X_{k+1}\right)\right\} - \mathrm{E}\left\{V\left(X_k\right)\right\} + \mathrm{E}\left\{z_k^T z_k\right\} - \varphi^2 \mathrm{E}\left\{w_k^T w_k\right\} \tag{16}$$

Which can rewritten as:

$$\mathrm{E}\left\{\begin{bmatrix} X_k \\ w_k \end{bmatrix}^T \Lambda \begin{bmatrix} X_k \\ w_k \end{bmatrix}\right\} \tag{17}$$

Where:

$$\Lambda = \begin{bmatrix} \Lambda_1 & \Lambda_2{}^{\mathrm{T}} \\ \Lambda_2 & \Lambda_3 \end{bmatrix}$$

$$\begin{aligned} \Lambda_1 = &\left(\bar{A} + \bar{\Delta}_A\right)^T P\left(\bar{A} + \bar{\Delta}_A\right) + \left(\bar{\alpha} - \bar{\alpha}^2\right)\left(\bar{B} + \bar{\Delta}_B\right)^T P\left(\bar{B} + \bar{\Delta}_B\right) \\ &+ \left(\bar{\beta} - \bar{\beta}^2\right)\bar{A}_\beta{}^T P\bar{A}_\beta + \bar{D}^T\ \bar{D} - P \end{aligned}$$

$$\Lambda_2 = \bar{B}_w{}^T P\left(\bar{A} + \bar{\Delta}_A\right) + \left(\bar{\beta} - \bar{\beta}^2\right)\bar{B}_\beta{}^T P\bar{A}_\beta$$

$$\Lambda_3 = \bar{B}_w{}^T P\bar{B}_w + \left(\bar{\beta} - \bar{\beta}^2\right)\bar{B}_\beta{}^T P\bar{B}_\beta - \varphi^2 I$$

The function in (16) is definite negative if:

$$\Lambda = \begin{bmatrix} \Lambda_1 & \Lambda_2{}^{\mathrm{T}} \\ \Lambda_2 & \Lambda_3 \end{bmatrix} < 0 \tag{18}$$

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By applying Schur Complement on (18), we get the LMI (15). Hence, from (16), (17) and (18) we conclude that:

$$\mathrm{E}\left\{V\left(X_{k+1}\right)\right\}-\mathrm{E}\left\{V\left(X_k\right)\right\}+\mathrm{E}\left\{z_k^T z_k\right\}-\varphi^2\mathrm{E}\left\{w_k^T w_k\right\}<0 \quad (19)$$

By summing up (19) from 0 to ∞ we find:

$$\sum_{k=0}^{\infty}\mathrm{E}\left\{\left\|z_k\right\|^2\right\}<\varphi^2\sum_{k=0}^{\infty}\mathrm{E}\left\{\left\|w_k\right\|^2\right\}+\mathrm{E}\left\{V_0\right\}-\mathrm{E}\left\{V_\infty\right\} \quad (20)$$

Which achieves the proof.

$$\begin{bmatrix} -P & \bar{A}^T+\bar{\Delta}_A{}^T & \bar{B}^T+\bar{\Delta}_B{}^T & \bar{A}_\beta{}^T \\ \bar{A}+\bar{\Delta}_A & -P^{-1} & 0 & 0 \\ \bar{B}+\bar{\Delta}_B & 0 & -\left(\bar{\alpha}-\bar{\alpha}^2\right)^{-1}P^{-1} & 0 \\ \bar{A}_\beta & 0 & 0 & -\left(\beta-\bar{\beta}^2\right)^{-1}P^{-1} \end{bmatrix}<0 \quad (10)$$

$$\begin{bmatrix} -P & 0 & \left(\bar{A}+\bar{\Delta}_A\right)^T P & \sqrt{}_\alpha\left(\bar{B}^T+\bar{\Delta}_B{}^T\right)P & \sqrt{}_\beta\,\bar{A}_\beta{}^T P & \bar{D}^T \\ 0 & -\varphi^2 I & \bar{B}_w{}^T P & 0 & \sqrt{}_\beta\,\bar{B}_\beta{}^T P & 0 \\ P\left(\bar{A}+\bar{\Delta}_A\right) & Q\bar{B}_w & -P & 0 & 0 & 0 \\ \sqrt{}_\alpha\,P\left(\bar{B}+\bar{\Delta}_B\right) & 0 & 0 & -P & 0 & 0 \\ \sqrt{}_\beta\,P\bar{A}_\beta & \sqrt{}_\beta\,P\bar{B}_\beta & 0 & 0 & -P & 0 \\ \bar{D} & 0 & 0 & 0 & 0 & -I \end{bmatrix}<0 \quad (15)$$

C. The uncertainty transformation

Based on the obtained result of theorem 1 and theorem 2, and the S-procedure theorem in [7] we can easily deduce the LMI (21).

In order to derive the controller parameters, we apply a suitable congruence transformation of (21). For this goal, the matrix P is partitioned as:

$$P=\begin{bmatrix} E & G \\ G^T & \varnothing \end{bmatrix}=\begin{bmatrix} F & J \\ J^T & \varnothing \end{bmatrix}^{-1}>0 \quad (22)$$

Where E, F are $\ell\times\ell$ symmetric positive-definite matrices; G, J are non-singular matrices and $\varnothing$ denotes matrix block that is irrelevant for the derivation of controller parameters.

From $PP^{-1}=I$ we conclude that the transformation matrices

$$\Psi_F=\begin{bmatrix} F & I \\ J^T & 0 \end{bmatrix},\ \Psi_E=\begin{bmatrix} I & E \\ 0 & G^T \end{bmatrix}$$

Satisfy the following conditions:

$$P\Psi_{\mathrm{F}}=\Psi_{\mathrm{E}};\ \Psi_{\mathrm{F}}{}^{\mathrm{T}}P=\Psi_{\mathrm{E}}{}^{\mathrm{T}} \quad (23)$$

We performed a first congruence transformation of (21) with $\Gamma=diag\left(\Psi_F,I,\Psi_F,\Psi_F,\Psi_F,I,I,I\right)$ on the right and Γ^T on the left with considering equalities (23). A second congruence transformation with $\Gamma=diag\left(F^{-1},I,I,I,E^{-1},I,E^{-1},I,E^{-1},I,I,I\right)$ is then applied on the right and on the left, to give the LMI (24).

Where: $\begin{cases}\Gamma_1=C_{ctr}J^TF^{-1},\ \Gamma_2=ZGA_{ctr}J^TF^{-1} \\ \Gamma_3=ZGB_{ctr},\end{cases}$

And $Z=E^{-1}$ (25)

The feasibility of (24) is conditioned by the non-singular matrices J, G chosen such that $GJ^T=I-EF$

Consequently, the controller parameters in (7) can be deduced as follows:

$$\begin{cases} A_{ctr}=G^{-1}Z^{-1}\Gamma_2\left(ZF^{-1}-I\right)^{-1}ZG \\ B_{ctr}=G^{-1}Z^{-1}\Gamma_3 \\ C_{ctr}=\Gamma_1\left(ZF^{-1}-I\right)ZG \end{cases} \quad (26)$$

By applying a linear transformation $\tilde{x}_k=ZG\hat{x}_k$, we get a new representation of the controller (7):

$$\begin{cases} \tilde{x}_{k+1}=\tilde{A}_{ctr}\tilde{x}_k+\tilde{B}_{ctr}y_k \\ u_k^{ctr}=\tilde{C}_{ctr}\tilde{x}_k \end{cases} \quad (27)$$

Where:

$$\begin{cases} \tilde{A}_{ctr}=\Gamma_2\left(ZF^{-1}-I\right)^{-1} \\ \tilde{B}_{ctr}=\Gamma_3 \\ \tilde{C}_{ctr}=\Gamma_1\left(ZF^{-1}-I\right) \end{cases}$$

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$$\begin{bmatrix} -P & 0 & \bar{A}^T P & \sqrt{}_{\alpha}\, \bar{B}^T P & \sqrt{}_{\beta}\, \bar{A}_{\beta}{}^T P & \bar{D}^T & 0 & \lambda I \\ 0 & -\varphi^2 I & \bar{B}_w{}^T P & 0 & \sqrt{}_{\beta}\, \bar{B}_{\beta}{}^T P & 0 & 0 & 0 \\ P\bar{A} & P\bar{B}_w & -P & 0 & 0 & 0 & \rho_a P & 0 \\ \sqrt{}_{\alpha}\, P\bar{A}_{\alpha} & 0 & 0 & -P & 0 & 0 & \rho_b P & 0 \\ \sqrt{}_{\beta}\, P\bar{B} & \sqrt{}_{\beta}\, P\bar{B}_{\beta} & 0 & 0 & -P & 0 & 0 & 0 \\ \bar{D} & 0 & 0 & 0 & 0 & -I & 0 & 0 \\ 0 & 0 & \rho_a P & \rho_b P & 0 & 0 & -\lambda I & 0 \\ \lambda I & 0 & 0 & 0 & 0 & 0 & 0 & -\lambda I \end{bmatrix} < 0 \tag{21}$$

$$\begin{bmatrix} -F^{-1} & * & * & * & * & * & * & * & * & * & * & * \\ -F^{-1} & -E & * & * & * & * & * & * & * & * & * & * \\ 0 & 0 & -\varphi^2 I & * & * & * & * & * & * & * & * & * \\ A_1 + B_1\Gamma_1 & A_1 & B_2 & -F & * & * & * & * & * & * & * & * \\ A_1 + \Gamma_3 C_1 + B_1\Gamma_1 + \Gamma_2 & A_1 + \Gamma_3 C_1 & B_2 + \Gamma_3 C_2 & -Z & -Z & * & * & * & * & * & * & * \\ \sqrt{}_{\alpha}\left(A_{\alpha} - B_{\alpha}\Gamma_1\right) & \sqrt{}_{\alpha}\, A_{\alpha} & 0 & 0 & 0 & -F & * & * & * & * & * & * \\ \sqrt{}_{\alpha}\left(A_{\alpha} - B_{\alpha}\Gamma_1\right) & \sqrt{}_{\alpha}\, A_{\alpha} & 0 & 0 & 0 & -Z & -Z & * & * & * & * & * \\ \sqrt{}_{\beta}\, A_{\beta} & \sqrt{}_{\beta}\, A_{\beta} & \sqrt{}_{\beta}\, B_{\beta} & 0 & 0 & 0 & 0 & -F & * & * & * & * \\ \sqrt{}_{\beta}\left(A_{\beta} + \Gamma_3 C_{\beta}\right) & \sqrt{}_{\beta}\left(A_{\beta} + \Gamma_3 C_{\beta}\right) & \sqrt{}_{\beta}\left(B_{\beta} - \Gamma_3 C_W\right) & 0 & 0 & 0 & 0 & -Z & -Z & * & * & * \\ D_d & D_d & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -I & * & * \\ 0 & 0 & 0 & \rho_a F & \rho_a Z & \rho_b F & \rho_b Z & 0 & 0 & 0 & -\lambda I & * \\ \lambda I & \lambda I & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\lambda I \end{bmatrix} < 0 \tag{24}$$

We proceed to turn all constraints into LMI (24), for that we note:

$$S = F^{-1} \tag{28}$$

To solve the obtained LMI, we have to solve the problem below:

$$\min_{\textit{Subject to } (24),\ (25)\ \textit{and}\ (28)} \varphi \tag{29}$$

For the variable φ, *and the symmetric positive definite matrices* $E,\ F,\ S,\ Z, \Gamma_1, \Gamma_2, \Gamma_3$ *and the positive scalar* γ

To solve the abovementioned non-convex optimization problem, we have adopted the SLPMM (Sequential Linear Programming Matrix Method) [8] and the SDP (Semi-Definite Programming) for the relaxation of the constraints:

Then $S = F^{-1}$ and $Z = E^{-1}$ holds if and only if: $\begin{bmatrix} S & I \\ I & F \end{bmatrix} \succ 0$, $\begin{bmatrix} Z & I \\ I & E \end{bmatrix} \succ 0$, $Trace(SF) \geq \ell$ and $Trace(ZE) \geq \ell$ hold.

The non-convex problem (29) can be resolved by finding a feasible solution for the above problem:

$$\min_{\textit{Subject to } (24)\ (25)\ \textit{and}\ (28)} \left(Trace(SF) + Trace(ZE) + \varphi^2 \right) \tag{30}$$

For the variable φ, *and the symmetric positive definite matrices* $E,\ F,\ S,\ Z, \Gamma_1, \Gamma_2, \Gamma_3$ *and the positive scalar* γ

We propose the following ILMI (iterative LMI) algorithm to solve (30) in order to achieve the computation of the parameters of the studied controller:

1- Find the initial point E^0, F^0, S^0, Z^0 such that the LMIs (24) hold and set $\varphi_{\min} = \varphi$

2- Find $E^k, F^k, S^k, Z^k,\ \Gamma_1, \Gamma_2, \Gamma_3, \lambda$ by solving:

$$\min_{\textit{Subject to } (24)\ (25)\ \textit{and}\ (28)} \left(Trace\left(SF^k + S^k F\right) + Trace\left(ZE^k + Z^k E\right) + \varphi^2 \right)$$

3 - If

$$\left| Trace\left(SF^k + S^k F\right) + Trace\left(ZE^k + Z^k E\right) + \varphi^2 \right| \leq 4(n + m + p)$$

then $\varphi_{\min} = \min\{\varphi, \varphi_{\min}\}$. The solution is $E,\ F,\ S,\ Z,\ \Gamma_1,\ \Gamma_2,\ \Gamma_3,\ \lambda,\ \varphi_{\min}$ and compute the controller parameters (26)

Else k= k+1 then go to step 2, until finding a feasible solution.

V. SIMULATION RESULT

We consider a numerical example to validate the effectiveness of the presented theoretical results. System parameters are chosen as:

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$$A=\begin{bmatrix}0.5 & 0.2 & 0.45\\ 0.1 & -0.5 & 0.2\\ 0.24 & 0 & 0.1\end{bmatrix}, B=\begin{bmatrix}2 & 1\\ 0.5 & 1.6\\ 0.13 & 1.5\end{bmatrix}, C=\begin{bmatrix}0.8 & 0.2 & 0.4\\ 0.1 & 0.1 & 0.1\end{bmatrix}$$

$$B_W=\begin{bmatrix}0.3\\ 0.1\\ 0.3\end{bmatrix}, C_W=\begin{bmatrix}0.3\\ 0.1\end{bmatrix}, D=\begin{bmatrix}0.1 & 0 & 0\end{bmatrix}, \rho_a=0.5, \rho_b=0.1$$

The simulation are performed using MATLAB YALMIP [9], the SeDuMi solver [10] and the Truetime simulator [11].

For stochastic variable, α et β we note $\bar{\alpha}=\bar{\beta}=0.1$, with 10^{-3} accuracy, the $H\infty$ performance is $\varphi_{\min}=0.06$, this result is obtained in 130 iterations.

The closed loop state trajectories under wireless network ZigBee (IEEE 802.15.4) are illustrated in the Figure 1.

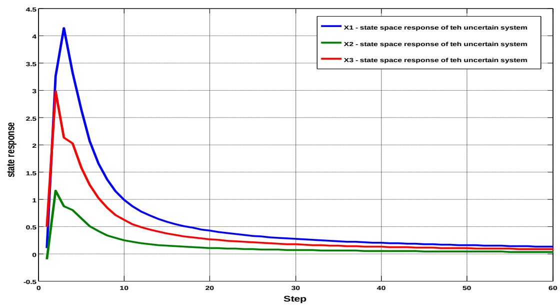

Fig. 1. The closed loop state trajectories of the stochastic uncertain Networked Control System over wireless network

It is obvious from the state space trajectories of the closed loop system that the system is exponentially mean square stable, which prove the validity of the adopted approach.

VI. Conclusion

This paper proposes the design of an $H\infty$ dynamic feedback controller for a class of uncertain linear discrete-time system that is subject to (1) stochastic packet loss in both control and measurement channels and (2) norm-bounded uncertainties in both state and control matrix. The exponential stability is studied in the sense of mean square convergence. Simulation results via numerical example shows the behaviour of the WNCS and have demonstrated the effectiveness of the proposed approach.

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